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The fractal mind of pedologists (soil taxonomists and soil surveyors)

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ABSTRACT

There has been little work in science dealing with the organizational, political and scientific layering of database structures as well as classifications and surveys of natural resources. There is disagreement among scientists whether taxonomies are invented (human-made constructs) or are discovered ("natural" structures) independent of the discipline involved. We believe it would be helpful to study the nature of taxonomies from different points of view in order to examine questions such as; are there common features in all taxonomic systems?, are the systems neutral?, and how are classifications and data collection (surveys) linked? It is generally accepted that much institutional work on soil classification systems was nationally biased, especially in terms of practical land management.

Recent studies show that the USDA soil taxonomy has the same mathematical structure as some biological ones that conform to physical laws that dictate and optimize information flow in user friendly retrieval systems. In this paper we demonstrate that the multifractal nature of the USDA soil taxonomy is strongly linked with conventional soil survey practices. In fact most surveys are packed with power law distributions, such as: (i) hierarchic taxonomic level used according to the scale map; (ii) minimum polygon size fits the functions to the map scale; and (iii) boundary density–scale map relationship, among others [Beckett, P.H.T., Bie, S.W., 1978. Use of soil and land-system maps to provide soil information in Australia. CSIRO Aust. Div. Soils, Technical Pap. No. 33, pp. 1–76]. Consequently a plethora of power law examples appear in soil survey products and soil taxonomies. Because both activities are strongly linked it seems the minds of soil surveyors and soil taxonomists create similar fractal structures. Fractal objects and power laws are scale invariant mathematical constructs, and the products prepared by experts are also fractal in many aspects. This process could be the reason that maps devoid of legends and other information have a high resemblance and information content, and with independence of scales, they provide a clear fractal signature.

In summary, the systems used by soil surveyors and soil taxonomists as a whole have fractal-like structures. We now believe that developing and using fractal structures are subconscious activities of the human brain reflecting both nature and our way of processing and representing information. Because the standards of many natural resource maps are similar to pedological ones, we suspect that scale-invariant information processing is intuitive to human beings and that a more rigorous formalization of survey-taxonomy architectures may help practitioners better understand their activities and constructs, and provide a way to improve them.

1. Introduction

1.1. Mathematical structures

Much has been published about taxonomies and cartographic standards in general and on soil classifications, surveys and maps in particular. Most articles have been about computer assisted applications. In the case of taxonomies, many focus on managing and classifying millions of documents on the Internet or in

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libraries. Publications about classical maps and taxonomy standards are scarcer and information about mathematical structures of taxonomies is rather meagre. It is intriguing that comparisons about the mathematical structures of soil taxonomies and cartographic representations may reveal, in part, how the human mind works (Ibáñez and Ruiz-Ramos, 2006; Ibáñez et al., 2006a). For soil taxonomists, surveyors and cartographers it is interesting to understand their tasks from a formal mathematical point of view and this invites comparisons with procedures utilized in other related disciplines. It seems, for example, that all maps devoid of legends and other information have a high resemblance and information content, and with independence of scales, there is a clear fractal signature. This may mean that pedologists unconsciously use self-similar or scale invariant mental procedures. If so, it should be possible to formalise these from a mathematical point of view.

Currently, fractals (Mandelbrot, 1983) and multifractal tools (Feder, 1988) are being applied to better understand a plethora of natural and cultural products. Fractals and multifractals inform about possible scale invariant structures of the phenomena studied. It is noteworthy that the mathematical structure of biological classifications has not been given attention by more scientists, with some interesting exceptions (Willis, 1922; Willis and Yule, 1922; Burlando, 1990, 1993; Minelli et al., 1991; Minelli, 1993). Recently soil taxonomies have begun to be studied in this context (Guo et al., 2003; Ibáñez and Ruiz-Ramos, 2006; Ibáñez et al., 2006). Some authors have used power law, fractal, and multifractal formalisms to compare pedological and biological taxonomies, thereby summarising many of these inquiries (Ibáñez and Ruiz-Ramos, 2006; Ibáñez et al., 2006).

1.2. Cartographic concepts

Maps have been used for centuries to visualize spatial data and help users better understand spatial relationships (Kraak and Ormeling, 1996). Every map is an abstraction of reality and presents information in a synoptic manner. Cartographers have studied cartographic communication processes and suggested different models to explain them. Some very well known models, such as Kolacny's model and the Robinson and Bartz-Petchenik model, can be found in Bos (1984). In order to guarantee the success of the communication process, a cartographer should elaborate or construct a map using well designed rules and symbols that reflect the main characteristics and spatial relations of the geographical phenomena. These relations should easily be perceived by the users when reacting to the visual impressions created by the symbols (Bos, 1984). Symbols, as the minimum unit of knowledge, should be combined properly following certain perceptual rules. These rules are collectively known as "Graphic Semiology" and constitute the "grammar of cartographic language" (Bertin, 1981, 1984). In map planning, scale is a critical specification. Scale determines the size of the map sheet for a given ground area, the accuracy needed in the surveys, and the amount of detail that can be represented on the map as well as directly affecting the cost and rate of progress of the work. The trend has been toward the use of larger scales, reflecting the need by map users for more detailed information about the land surface.

A map series is set of maps that conform generally to the same specifications and cover an area or a country in a systematic pattern. The maps of a series have the same scale, format and system of symbolization. When making these specifications cartographers put the emphasis on two aspects: accuracy standards and minimum sizes of the symbols. National map agencies in each country commonly have sets of map series (for example, topographic series) at different middle and small scales, based on their larger scale maps, and proceeding to smaller scale maps through processes of scale reduction and generalization. When changing map scale the relative space available for the portrayal of the phenomena is obviously a function of scale and it is important to keep in mind that the reduction of available space takes place as the square of the ratio of the difference in linear scales. This means reducing complexity and details to illustrate overall trends and main ideas.

Map generalization is a very complex visual–intellectual process. Robinson (1984, in Pérez-Gómez, 2003) defines the whole process in terms of four aspects: simplification, classification, symbolization and induction. Traditional map generalization was rather subjective until the widespread use of computer cartography and Geographic Information Systems in the 1980s. Since then computer algorithms have been used to evaluate computer generalization schemes (Pérez-Gómez, 2003) and the process of generalization has become more objective. Other authors classify these processes in terms of conceptual generalization and graphic generalization (Hole and Campbell, 1985; Muller, 1991; Kraak and Ormeling, 1996). In the broad field of map generalization, fractals have occasionally been used in the development of some line generalization algorithms such as the one described by Muller (1987). When reviewing different areas of cartography, we found no reference related to fractal structures or power fit formulas that guide the content and production of maps.

Beckett and Bie (1978) showed that many soil maps have attributes that conform to power law distributions including (i) map scale-area surveyed, (ii) standard line density-scale dependency, (iii) sampling density-mapped area, (iv) hierarchic taxonomic level used according to the scale map, (v) minimum polygon size fitting the functions on the map scale, (vi) soil survey effort depending of the scale map, and (vii) boundary density–scale map relationships.

Experts in cartography know more or less well the rules that they apply. However in the bibliographic material consulted there were no references about implications of human cognitive bias in this practice, or to mathematical relationships between classifications and map scale. Although not many documents explicitly mention the latter topic (USDA-Soil Survey Staff, 1993; Rossiter, 2000), in general, these publications are in agreement with the plots shown by Beckett and Bie (1978). Hupy et al. (2004) examined recently deglaciated soilscapes in the USA to determine how the scale of existing paper soil maps affects the amount of information that can be portrayed. They noted, for 13 counties using either whole county data or sampled data, very high R^2 values of power law fits for punctuate map units km^{-2}, polygons km^{-2}, polygon boundary length (m) km^{-2}, and polygon boundary length (m) $polygon^{-1}$ km^{-2}.

A statistical analysis of the best fits of the "guide to map scales and minimum delineation size" of the USDA Soil Survey Manual (Table 2-2, p. 53) detected near perfect correlations to power laws and quadratic distributions. Some of Dent and Young's (1981) recommendations (Table 6.1, p. 90) also fit well to a linear function. Although quadratic functions have been recognised for a long time in cartography, this is not so evident for power laws. These statistical distribution models may be formalized as power law ($y = ax^b$), quadratic fit ($y = a + bx + cx^2$) and linear fit ($y = a + bx$).

It is clear that, depending on the scale and purposes of a map, the various classes and sub-classes to be represented will be expressed in various degrees of detail. In thematic maps, such as soil maps, the principal subject is represented in detail by including a large range of sub-classes. Thus, the definition of the classes of features represented by the symbols is just as much part of the map accuracy as putting them in the right place on the map. Defined classes and sub-classes of map features should be consistent and used systematically. Pedologists attempt to do this when applying taxonomy and other guidelines when surveying soil landscapes. Dent and Young (1981) associated map scales with different kinds of soil cartographic units and the US Soil Survey Manual (USDA, SSDS, 1993, p. 48) has a key for identifying kinds of soil surveys.

In general, all land thematic maps need a topographic base map, thus it was once common for natural resource maps (e.g. soils) to utilize existing topographic maps and there emerged a clear relation between maps and classifications to cover the area of a given country, area or region in a systematic pattern.

1.3. Working hypothesis

Our main objective in this work was to test our working hypothesis that pedologists who design soil taxonomies and those who make soil survey maps unconsciously apply logic in such a way that their products conform to fractal structures. Because power law distributions are scale invariant, we believed an analysis of distributions of ST taxa and soil maps using these taxa, taxonomic information in such maps, and the size-number frequency of delineations might reveal fractal structures. We suspect that other types of natural resource inventories might also complement our findings.

2. Materials and methods

2.1. Natural resource maps

Most soil classification schemes have been designed to support practical purposes: commonly, land use and land management related to agriculture and forestry. The scales of maps depicting soil resources have been consistent with the size of the areas for which information was desired; general regional planning needed only broad generalized areas suitable for major kinds of land uses, whereas individual small farms need much more detailed information to guide management practices such as crop suitability, productivity estimates, drainage, etc.

In the 1970s, Beckett and his students began to examine soil surveys to determine if they had common characteristics. In a seminal paper, Beckett and Bie (1978) described maps from Australia and observed that many features were linear when plotted as log–log relationships. They noted that, in general, compound units became more complex (more classes in each unit) as the map scale decreased (Fig. 1). They plotted the number of great groups, soil types, and soil series versus size of area surveyed as well as map scale versus size of area surveyed. In larger scale surveys they plotted the taxonomic level of the dominant class in a map unit versus size of area surveyed. Recent studies show that pedodiversity-area and biodiversity-area relationships commonly conform to power law distributions (Ibáñez et al., 2005a,b).

Beckett and Bie also quantified the intricacy of soil patterns mapped at any scale, such as length of boundary on map (cm) versus area of map (cm^2) and found similar relationships (Rossiter, 2000) They also studied the costs and effort of conducting field soil surveys noting again power law fits to the data.

For many years the National Cooperative Soil Survey in the US produced soil surveys in agriculturally important counties at a scale of 1:15,840 (4 in. = 1 mile) and 1:20,000 (~3.2 in. = 1 mile) for ease in planning individual farms platted in the latitude/longitude system of land survey. These soil surveys were done as free surveys where the number of transects or observations were not required, but guidelines specified that soil boundaries should be observed in their entirety, thus relying heavily on aerial photography and/or satellite imagery to extrapolate boundaries (SSM, 1951). Rossiter (2000) reported the frequency of ST categories in the detailed soil survey of Edgecombe County as 4 Orders, 8 Suborders, 12 Great Groups, 21 Subgroups, 35 Families, 38 Soil Series, and 52 map units

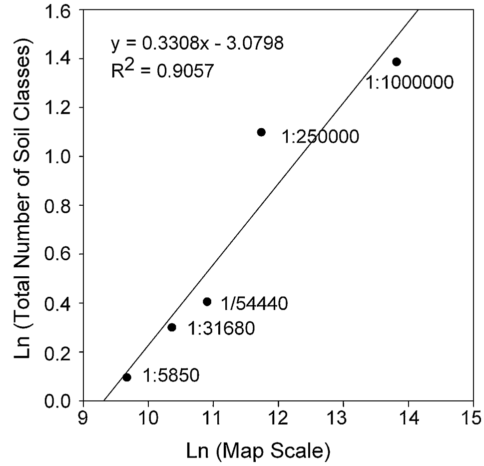

Fig. 1. Number of soil classes in soil associations as dependent on map scale. Data from Beckett and Bie (1978). Regression line: $y = 0.33x + 3.08$, $R^2 = 0.91$.

that are mainly phases of soil series. A log–log plot with the taxonomic levels shown as increasing levels within a hierarchy is provided in Fig. 2.

A variety of survey areas representing the West (Umatilla and Klamett counties, Oregon; Nye county, Nevada), Midwest (Tama and Pocahontas counties, Iowa), South (Jefferson county, Alabama), and East (Windsor county, Vermont) having a range of parent materials, climate, age, vegetation and land use were examined for mathematical patterns (see Figs. 3 and 4).

A compilation of areas of each soil Order of ST by Guo et al. (2003) enabled us to examine the relationships between areas and the number of subtaxa. That is, area of a soil Order/no. of suborders in that Order, area of a soil Order/no. of great groups in that Order, and area of soil Order/no. of soil series in that Order. The number of soil series in the area of each soil Order is shown in Figs. 5 and 6.

The European Soil Database v.2.0 contains a digitized soil map of the European continent at 1:1 M scale (European Soil Bureau, 2004) with units identified with the second level of WRB. The harmonization of the independent country products was difficult due to different pedological backgrounds and mapping intensities and is rather heterogeneous. The fit of number of polygons with the area covered by these polygons is shown in Fig. 7.

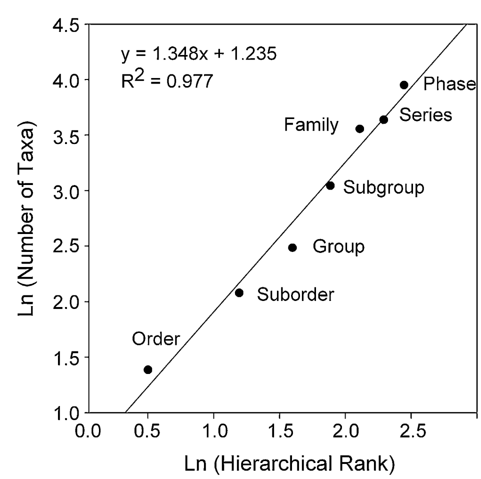

Fig. 2. Number of taxa for all hierarchical levels of the USDA Soil Taxonomy in Edgecombe County, North Carolina, EE. UU. Data from Rossiter (2000). Regression line: $y = 1.35x + 1.24$.

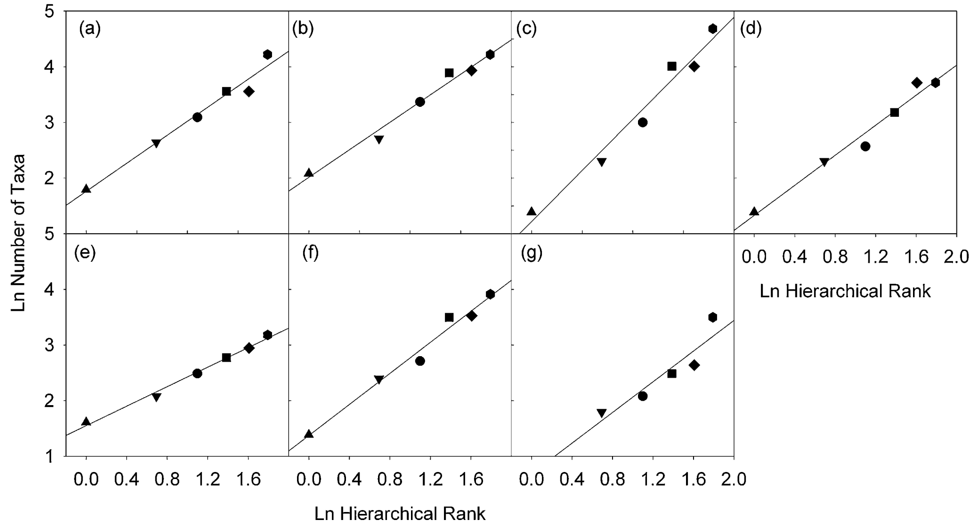

Fig. 3. Dependencies between the number of taxa and their hierarchical rank for seven US counties. a—Umatilla County, b—Klamath County, c—Nye County, d—Tama County, e—Jefferson County, f—Windsor County, g—Pocahontas County; (▲) order, (▼) suborder, (●) group, (■) subgroup, (◆) family, (●) series. Linear regression lines in the log–log scale: a—$y = 1.259x + 1.761$, $R^2 = 0.972$; b—$y = 1.235x + 2.012$, $R^2 = 0.982$; c—$y = 1.835x + 1.218$; $R^2 = 0.969$; d—$y = 1.345x + 1.330$, $R^2 = 0.973$; e—$y = 0.877x + 1.550$, $R^2 = 0.992$; f—$y = 1.394x + 1.376$, $R^2 = 0.980$; g—$y = 1.378x + 0.686$, $R^2 = 0.946$.

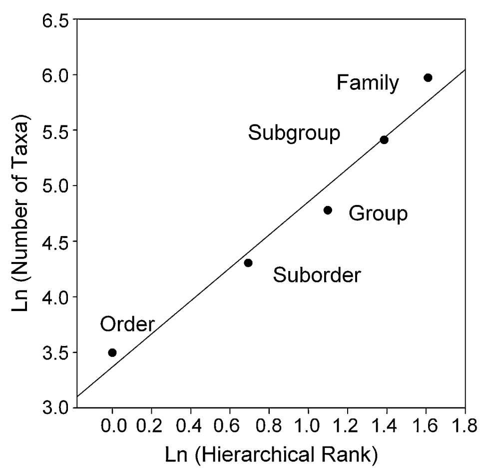

Fig. 4. Dependencies between the number of taxa and their hierarchical rank for combined data from the seven county soil surveys. The linear regression line in the log–log scale is $y = 1.487x + 3.369$, $R^2 = 0.968$.

A potential vegetation map of the Iberian Peninsula at 1:1 M using GIS tools (Carrera, 1998) and a FAO soil map (Food and Agriculture Organization, 1980) of the same region were available to examine spatial characteristics. The frequencies of drainage basins have been compiled by Horton–Strahler Ranks and by log-size-frequencies. Diversity indices by drainage basin size of both soils and vegetation are shown in Fig. 8a and 8b.

2.2. Power law distributions

Although quadratic fits, $y = a + b + cx^2$, are well known by cartographers, and linear fits, $y = a + bx$, are common in almost all scientific fields, less has been reported about power law fits, $y = ax^b$. Our main interest in power law distributions is that they are scale independent, thus their occurrence may suggest that fractal or multifractal systems provide a framework for comprehending portions of natural resource inventories and surveys. It is common to test for power law fits by plotting the parameters as log–log, or ln–ln relationships and observing their fit to straight lines. Several other regression models could fit nicely to the same datasets, thus it is difficult to select the best when the data are not very numerous (Korvin, 1992). Because this a pervasive problem (Mitzenmacher, 2004; Buchanan, 2008) and guidelines for soil map

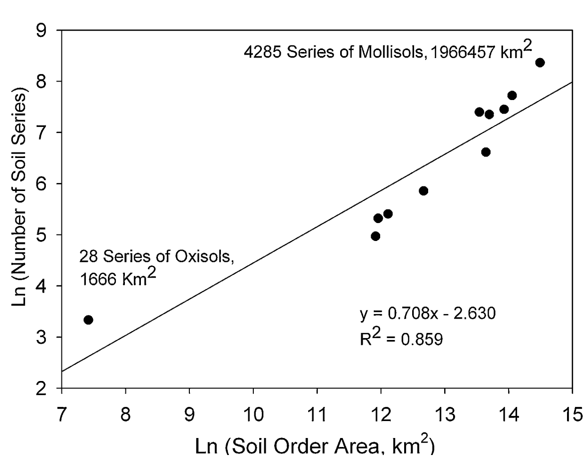

Fig. 5. Dependency of the number of soil series in soil orders on the area occupied by the orders in the United States. Data from Guo et al. (2003).

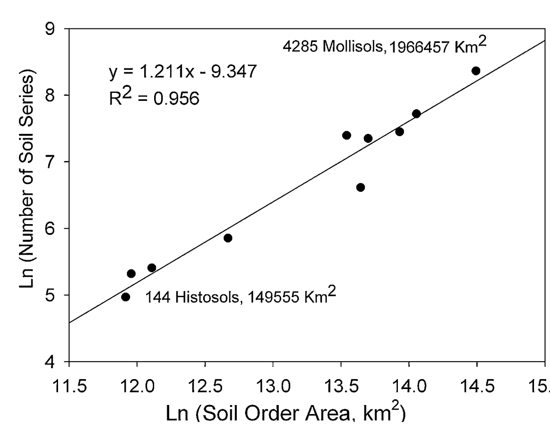

Fig. 6. Dependency of the number of soil series in all soil orders but Oxisols on the area occupied by the orders in the United States data from Guo et al. (2003).

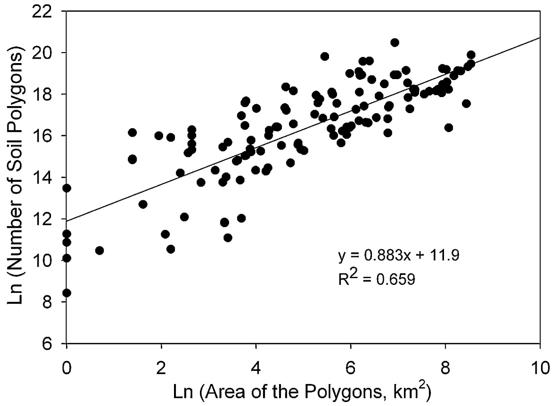

Fig. 7. Relationship between the number of soil polygons at the soil map of Europe and the size of these polygons. Map scale 1:1,000,000, data from the European Soil Bureau (2004).

delineations and contents of the according to the scale follow a straight line in a log–log plot (USDA-Soil Survey Staff, 1993; Rossiter, 2000) we consider that a close fit to a power law likely supports our hypothesis, although in some instances regression models such as a polynomial may provide a better fit.

2.3. Diversity indices

For many scientists the concept of diversity is restricted to "richness", the number of different taxa present in a certain ecosystem. However, the proportional abundance of each taxon is the most frequent way of estimating diversity (Ibáñez et al., 1990, 1995). From this point of view, pedodiversity may be divided into two elements: "richness" is the number of soil taxa according to the classification in use; "evenness" refers to the relative abundance of each taxon. Diversity indices most used in ecology come from Information Theory (Magurran, 1988) and Shannon's Index is often preferred for its ease of calculation.

Diversity can be equated with the amount of uncertainty of collecting different taxa of a given sampled area (population). The more taxa there are, and the more even their representation, the greater the uncertainty and hence the greater the diversity. Information content, which is a measure of uncertainty, is therefore a reasonable measure of diversity (Margalef, 1958).

Shannon's Index has very close mathematical connections to Boltzmann's measure of entropy of thermodynamic systems. In fact they are equivalent and that is why Shannon called it an Entropy Index. Its mathematical expression is

$$H' = -\sum_{i=1}^{n} p_i \ln p_i \tag{1}$$

where H' is the negative entropy, "negentropy", or diversity of the system, and p is the proportion of individuals found in the ith taxon. In calculating Shannon's Index, any logarithmic base can be adopted. The units of H' are the same as in Information Theory (Pielou, 1969). Thus, the value of H' is the sum of the proportions of the individual objects multiplied by the negative logarithm of the proportion. It ranges from 0 (ln of 1) if all of the individuals are of one taxon, to $\ln N$ if the number of taxa equals the number of individuals. The index is maximum for any richness if all taxa have equal numbers of individuals and minimum if the individuals are maximally concentrated in one taxon. This index is a measure of information for a group of taxa (species, soil types, etc.) which have different probabilities of being represented, i.e. different numbers of individuals. Information is maximum when the probabilities (proportional abundances) of all taxa are equal. Information is 0 if there is only one possibility, i.e., diversity is 0.

3. Results and discussion

We looked at several examples of relationships of soil classifications and soil maps to see if they were consistent with power law distributions. If so, such evidence would be supportive of fractal structures.

Although the categories of a hierarchy do not have numbers that can readily be expressed as logs, the descending levels are represented from 1 to 6 to illustrate a relationship (see Fig. 2). Thus for Edgecombe county, the number of taxa in each category versus the level (1–6) of the category is well characterized as a power law distribution.

Following with this approach, Fig. 3 shows similar results for seven detailed soil surveys of US counties. Regardless of the geomorphology, parent materials, climate and common vegetation, each of these surveys has a power law distribution of the number of taxa in each category of Soil Taxonomy. We believe that the variations of soil forming factors and their interactions result in different and unique soils within the pedosphere, consequently the similarity of spatial relationships demonstrates the validity of our working hypothesis. We would expect additional trials to exhibit similar patterns of power law distributions.

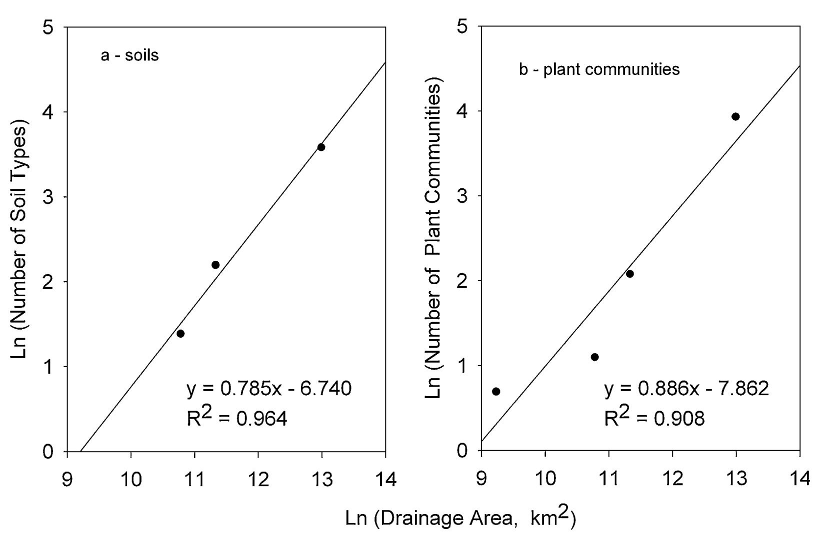

Fig. 8. Relationships between of the number of soil types (a) and plant communities (b) on area of the drainage basins at the Iberian Peninsula.

It is noted that although Pocahontas County appears to deviate from the general trend, it still has a strong correlation with a power law. The deviation indicates that there are more taxa in the higher categories (Orders and Suborders) than in the other surveys examined. There are numerous small punctuate soil units which are characteristic of the spatial complexity of recently deglaciated soilscapes (Hupy et al., 2004). Of interest also is the utilitarian or strong agricultural land use bias detected by the extra details mapped in this county where precision farming has become prominent. The average polygon is about 7.0 acres in size compared to the 25–30 acre sizes of polygons in general farming communities and the 200–250 acre sizes in mixed agriculture and grazing lands. A plot of the combined 7 county surveys in Fig. 4 shows the influence of the extra details available in the Pocahontas survey, however the coefficient of correlation remains high ($P < 0.01$).

Do the obvious variations of regions have little influence on the subtaxa within soil taxa, or is this a feature of the way pedologists interpret soil landscapes regardless of geology or land use? Is this nature or only the human response to nature? Is this a pattern of mental associations of information evolved through millennia of survival and growth development?

The data set of Guo et al. (2003) is shown in Fig. 5 as the number of soil series versus the area of the individual soil orders in the US. The Oxisols are the point of deviation indicating that more soil series are recognized for a small area than would be expected. Most Oxisols in the US are derived from basic rocks and represent only a small portion of Oxisols in the world. An emphasis on studying soils in tropical regions to enhance agrotechnology transfer was supported by the US Agency for International Development and resulted in refinements of the criteria, mainly moisture and temperature regimes, used to identify kinds of Oxisols. This in turn resulted in an increase of the number of US soil series belonging to the Oxisol Order. A re-plot without the Oxisols (Fig. 6) shows the strong relationship to a power law distribution.

Despite the diversity of pedological schools, details of survey information, and intensity of surveys throughout the European Union, there is still a remarkable resemblance to a power law fit (Fig. 7). We believe as additional harmonization of databases in the EU are obtained that the fit of taxa polygons versus areas will become closer. Although variations have existed (and continue to exist) among representations of the pedosphere, the remarkable aspect is the similarity of results irrespective of time and space and ideology. In many locations, human evolution is much shorter than that of a portion of the pedosphere raising the question about whether our thought processes and thought patterns are imposed on natural patterns, or that there are real patterns in nature that we perceive because of our evolution as humans. We do not know.

If we turn to other natural resource surveys and inventories, we note the very strong similarity of the various measures of diversity and evenness of soil and vegetation data for the Iberian Peninsula (Fig. 8a and b). Taxon diversity for vegetation and soils (Shannon Index) is quite similar. That is remarkable when you realize that different taxonomies were used, different land base maps were used, and information databases were not equal across the region. We believe that this is merely an example of what exists and will be detected in the natural resource surveys and inventories of the Earth. We see this as another source of support for our working hypothesis. We thought that divergence from power law fits would challenge our hypothesis, yet for the most part these examples tend to support our hypothesis that pedologists, cartographers, and other natural resource scientists respond in their interactions with nature as though their minds recognize, organize, and retrieve information in an efficient manner that is commonly characterized by power law distributions.

Cartozo et al. (2008) conducted an interesting study of taxonomic diversity using databases of several florae (collections of plant species populating specific areas) in different geographic and climatic regions. For each list of species they produced a taxonomic hierarchical classification and found that irrespective of geographic locations the flora had universal statistical properties relating them with power laws, with only minor deviations from power law distributions.

4. Conclusions

Soil classification schemes are mental products that humans use to organize a large body of information about kinds of soils so that many properties of the objects can be recalled fairly easily as well as providing relationships of groups and among groups of soils. In these aspects they are similar to schemes to classify other objects in the natural sciences, such as plants, animals, and landscapes. The structure of the USDA Soil Taxonomy can be represented by power law distributions and it is believed that Soil Taxonomy is a multifractal mathematical structure.

Field soil surveys are made at many scales. When aerial photography became readily available they commonly replaced topographic maps as the base maps on which to plot the polygons of specific kinds of soil map units. Smaller scale maps are generalized schematic soil maps whose map units are more complex because of associated and included soils present at larger scales but whose pattern of polygons is more simplified. A review of soil surveys in Australia by Beckett and Bie (1978) noted many log–log relationships among the features of taxa, polygons, and scales suggesting that power laws likely were a unifying feature of such natural resource maps.

Beckett and Bie (1978) mentioned that the complexity of map units (soil associations) increased as map scales decreased. A plot of their data (Fig. 1) fits well to a power law suggesting that in the field soils are perceived by surveyors to have a fixed scale; thus more combinations occur in soil associations as map unit polygons represent larger areas.

When the position of hierarchical levels of a taxonomy are represented by increasing numbers (1 for Orders, 2 for Suborders, 3 for Great Groups, and so forth) the number of soil taxa at each level in a detailed county survey as proposed by Rossiter (2000; Fig. 2) closely follows a power law distribution. Using the same approach, seven additional county surveys from different climatic and geologic regions in the US (Fig. 3, Table 1) reveal that the spatial

Table 1
Results of soil surveys for seven counties in the United States.

Orders	Suborders	Groups	Subgroups	Families	Series	Counties
6	14	22	35	39	68	Umatilla County Area, OR
8	15	29	49	51	68	Klamath County, OR, Southern Part
4	10	20	55	62	108	Nye County, NV, Northeast Part
4	10	13	24	28	41	Tama County, IA
2	6	8	12	14	33	Pocahontas County, IA
5	8	12	16	19	24	Jefferson County, AL
4	11	15	33	34	50	Windsor County, VT

Table 2
Results of fitting the power-law regression equation to the dependence of the number of taxa on the hierarchical level in Soil Taxonomy for seven counties in the United States.

Surveyed area	*a*	*b*	Standard error	R^2	*P*-value
Umatilla County Data Area Oregon	1.76	1.26	0.16	0.97	0.0003
Klamath County, Oregon, Southern Part	2.01	1.24	0.12	0.98	0.0001
Nye County, Nevada, Northeast Part	1.22	1.84	0.24	0.97	0.0004
Tama County, Iowa	1.33	1.35	0.17	0.97	0.0003
Pocahontas County, Iowa	0.69	1.38	0.24	0.95	0.0011
Jefferson County, Alabama	1.55	0.88	0.06	0.99	>0.0001
Windsor County, Vermont	1.38	1.39	0.15	0.98	0.0002

Legend: $y = ax^b$, where y = number of soil taxa of a given taxonomical level x = the hierarchical position of the taxa in the taxonomy [1 = orders; 2 = suborder, and so on].

Table 3
Statistics of drainage basins, Hortonian-Strahler rank-frequency distribution for Polygons, and Numbers of Plant and Soil Associations at 1 M Scale in Spain.

	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6
Soils						
Number of drainage basins	259	92	47	21	3	5
Area	75.6	216.7	769.2	3143	34,285	67,142
Number of soilscapes	3.4	5.83	11.21	33.00	190	341
Density (per area)	87.7	23.29	9.36	3.70	1.02	0.59
Shannon entropy[a]	0.62	0.89	1.10	1.18	2.43	2.66
Number of pedotaxa	2.64	3.62	5.11	8.43	31.00	39.20
Vegetation						
Number of drainage basins	259	92	47	21	3	5
Area	75.6	214.0	769.3	3142	34,285	67,142
Number of vegetation units	3.1	5.16	12.49	33.00	166.67	873.80
Density (per area)	95.5	20.85	9.15	3.70	0.79	1.17
Shannon entropy[a]	0.44	0.54	0.84	1.18	1.88	2.55
Number of vegetation units	2.46	3.02	4.70	8.43	18,67	71.00

[a] The Shannon entropy index is an index of diversity, $H' = -\sum p_i \times \ln p_i$, where H' is the negative entropy – negentropy – or diversity of the population, and p_i is the proportion of individuals found in this ith object. The value of p_i is estimated by n_i/N, where n_i is the number of individuals of the object considered, and N the total number of individuals collected (it may also be the percentage of area occupied by this ith object).

Table 4
Statistics of drainage basins, size-frequency distribution for polygons, and number of plant and soil, associations at 1 M scale in Spain.

Drainage basins area in km^2	<100		100–1000		1000–10,000		10,000–100,000	
	Vegetation types	Soil types	Vegetation types	Soil types	Vegetation types	Soil types	Vegetation types	Soil types
Number of drainage basins	229	228	159	160	31	31	8	8
Area of drainage basins	44.5	44.7	301.3	301.0	2.687.3	2687.3	54,820.2	54,820.2
Number of polygons	2.6	2.9	6.5	6.6	29.9	29.6	608.6	284.0
Polygons density	107.8	98.8	14.7	17.0	4.0	4.2	1.0	0.8
Shannon entropy[a]	0.4	0.6	0.6	0.9	1.2	1.3	2.3	2.6
Richness	2.3	2.4	3.4	3.9	8.2	8.5	51.4	36.1

[a] The Shannon entropy index is an index of diversity, $H' = -\sum p_i \times \ln p_i$, where H' is the negative entropy – negentropy – or diversity of the population, and p_i is the proportion of individuals found in this ith object. The value of p_i is estimated by n_i/N, where n_i is the number of individuals of the object considered, and N the total number of individuals collected (it may also be the percentage of area occupied by this ith object).

relationships of taxa in these survey areas also fit well to power laws. The fact that the *a* and *b* values of the power law fits differ for each survey area (Table 2) suggests that as landscapes and kinds of soils vary from place to place the nature of the power law distribution will also vary. There were many surveyors involved in mapping, maps were done at different times, and the training and background of the surveyors differed, yet the application of the taxa of Soil Taxonomy is remarkable.

Insofar as the mental construct of classification is multifractal and its application to field surveys appears to be scale invariant, the results support the idea that the logic in the design of Soil Taxonomy and the logic of pedologists mapping soils and identifying them with the taxa of that system are the same. Hupy et al. (2004) concluded that mapping soilscapes at larger scales enables more information to be added because the mapper is less constrained, or concerned with, minimum size of mapping units and it could be assumed that more time was available to produce larger scale maps. Do pedologsts think about such things while making soil surveys? We have no direct evidence that they do; rather it appears to be an unconscious application of logic whose products conform mathematically to fractal structures (Tables 3 and 4).

Overall the number of soil series (lowest taxonomic level in ST) in the area of each Order in the US fits a power law thus indicating that a fractal structure of soils in diverse landscapes is very likely. The new 1:1 M soil map of Europe is a collation of country maps at different scales, with different taxonomies, at different times, and with influences of different schools of pedology. The number of polygons on this map versus their area reveals a general tendency toward scale invariance. The disparity is due, we believe, to the use biases that vary widely throughout Europe.

The similarity of soil and vegetation patterns on the Iberian Peninsula is striking when considering that the maps were prepared for different purposes, using different methods and taxonomies. When the drainage basins were grouped by Rank (a branching phenomenon) or by log-sizes, the comparison of diversity (Shannon Index), richness, and the number of soil types and plant communities by basin size indicate that the way we see nature is often scale invariant. We adjust to scale differences with relative ease and if there are no legends to guide us, perceptions are that maps provide us with recognizable patterns with similar levels of comprehension. Is not the mind of a pedologist fractal?

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